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CONSTRUCTION OF EXPLICIT FUNCTIONS FOR DETERMINING THE COEFFICIENTS OF INTERNAL HEAT AND MASS TRANSFER FROM THE DATA OF MEASUREMENTS IN NONSTATIONARY REGIMES

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For a number of laws governing the variation of the characteristics of internal heat and mass transfer with respect to a spatial variable, we derive explicit functions relating them to the results of measurements of nonstationary temperatures or other potentials.

Many physical processes can be described by partial differential equations of the type represented by the nonstationary heat-conduction equation with coefficients which depend on a spatial variable. It is therefore of great practical interest to construct effective calculation algorithms whereby the data of measurements of some transfer potentials (for example, temperatures) can be used for estimating the parameters determining the character of the spatial variation of the coefficients involved. In some cases, exact explicit functions sufficiently suitable for practical realization can be obtained by using the method employed in [1, 2], namely, an analysis of the properties of the analytic solutions of the problem in the space of Laplace mappings.

In the case when it is permissible to describe a real process by a one-dimensional parabolic operator with coefficients dependent on a spatial variable, of the form

$$r^{-k}\frac{\partial}{\partial r}\lambda(r)r^{k}\frac{\partial T(r,\ \tau)}{\partial r}=c(r)\frac{\partial T(r,\ \tau)}{\partial \tau}, \tag{1}$$

where k = 0, 1, 2 for plane, cylindrical, and spherical fields, respectively, it is possible for a number of specific laws of variation of λ and c to obtain exact analytic solutions [3]. In particular, in the space of Laplace mappings a solution of the form

$$T(r,\ s)=\left(\frac{\lambda r^{k}cr^{k}}{E^{2}}\right)^{\frac{1}{2(m-2)}}\left\{AI_{-\frac{1}{m}}\left[\frac{2E\sqrt{s}}{m}\left(\frac{\lambda r^{k}cr^{k}}{E^{2}}\right)^{\frac{m}{2(m-2)}}\right]+BK_{-\frac{1}{m}}\left[\frac{2E\sqrt{s}}{m}\left(\frac{\lambda r^{k}cr^{k}}{E^{2}}\right)^{\frac{m}{2(m-2)}}\right]\right\} \tag{2}$$

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($I_{-1/m}$ and $K_{-1/m}$ are modified Bessel and MacDonald functions) is valid when λr^k and cr^k vary according to the laws

$$\tilde{\lambda}_0 r^l,\ \tilde{c}_0 r^n;\ \tilde{\lambda}_0(1+\alpha r)^l,\ \tilde{c}_0(1+\alpha r)^n;\ \tilde{\lambda}_0\exp(-lr),\ \tilde{c}_0\exp(-nr).$$

The parameters E and m are equal, respectively, to

$$E=\sqrt{\tilde{c}_0}\,\tilde{\lambda}_0^{\frac{m-1}{2}}(1-l)^{\frac{m-2}{2}},\quad m=\frac{n-l+2}{1-l};$$

$$E=\sqrt{\tilde{c}_0}\,\tilde{\lambda}_0^{\frac{m-1}{2}}\alpha^{\frac{m-2}{2}}(1-l)^{\frac{m-2}{2}},\quad m=\frac{n-l+2}{1-l};$$

$$E=\sqrt{\tilde{c}_0}\,\tilde{\lambda}_0^{\frac{m-1}{2}}l^{\frac{m-2}{2}},\quad m=\frac{l-n}{l}.$$

In the simplest case, when $\lambda(r)=\lambda_0 r^{l_0}$, $c(r)=c_0 r^{n_0}$:

$$T(r,\ s)=Ar^{-\frac{1}{2}(k+l_0-1)}I_{-\frac{1-(l_0+k)}{n_0-l_0+2}}\left[\frac{2r^{\frac{1}{2}(n_0-l_0+2)}}{n_0-l_0+2}\sqrt{\frac{s}{a_0}}\right]\tag{3}$$

for a plane layer, a solid cylinder, and a sphere;

$$T(r,\ s)=Br^{-\frac{1}{2}(k+l_0-1)}K_{-\frac{1-(l_0+k)}{n_0-l_0+2}}\left[\frac{2r^{\frac{1}{2}(n_0-l_0+2)}}{n_0-l_0+2}\sqrt{\frac{s}{a_0}}\right]\tag{4}$$

for a semibounded solid;

$$a_0=\lambda_0/c_0.$$

On the basis of formulas (3) and (4), the relation in the space of mappings between the temperatures at the points r = 0 and r = r_1 has the form

$$\varphi(s)=\frac{2r_1^{\frac{1-(l_0+k)}{2}}\Gamma(1-\nu)\,s^{\frac{\nu}{2}}}{(n_0-l_0+2)^{\nu}a_0^{\nu/2}}I_{-\nu}\left[\frac{2r_1^{\frac{n_0-l_0+2}{2}}}{n_0-l_0+2}\sqrt{\frac{s}{a_0}}\right]\tag{5}$$

or

$$\varphi(s)=\frac{2r_1^{\frac{1-(l_0+k)}{2}}s^{\frac{\nu}{2}}}{(n_0-l_0+2)^{\nu}a_0^{\nu/2}\Gamma(\nu)}K_{\nu}\left[\frac{2r_1^{\frac{n_0-l_0+2}{2}}}{n_0-l_0+2}\sqrt{\frac{s}{a_0}}\right];\tag{6}$$

$$\varphi(s)=T(r_1,\ s)/T(0,\ s);\quad \nu=\frac{1-(l_0+k)}{2+n_0-l_0}.$$

For this case, we consider the problem of determining the parameters n_0, l_0, a_0 from measurements of the temperatures T(0, τ), T(r_1, τ). From the form of (5) and (6) we can conclude that the ratio of the mappings $\varphi(s)$ = T(r_1, s)/T(0, s) is one of the particular solutions of the ordinary differential equation in the Laplace transform parameter

$$\varphi''_{zz}+\frac{1-2\nu}{z}\varphi'_z+\varphi=0,\tag{7}$$

where

$$z=\frac{2r_1^{\frac{n_0-l_0+2}{2}}\sqrt{s}}{(n_0-l_0+2)\sqrt{a_0}}.$$

After passing from $\sqrt{s}$ to s and from $\varphi(s)$ to its expression in terms of $T(r_1, s)$, $T(0, s)$, we find

$$s\{[T_1''(s)T_2(s)-T_1(s)T_2''(s)]T_2(s)-2T_2'(s)[T_1'(s)T_2(s)-T_1(s)T_2'(s)]\}+$$
$$+(1-\nu)T_2(s)[T_1'(s)T_2(s)-T_1(s)T_2'(s)]=\frac{r_1^{n_0-l_0+2}}{a_0(n_0-l_0+2)^2}T_1(s)T_2^2(s),$$
$$T_1(s)\equiv T(r_1, s),\quad T_2(s)\equiv T(0, s). \tag{8}$$

After making use of the theorem on the inversion of mappings in the space of originals we have

$$\psi(\tau)-\nu\psi_1(\tau)=d\varphi(\tau), \tag{9}$$

where

$$\psi(\tau)=\int_0^{\tau}(2\tau-3\bar{\tau})T_2(\tau-\bar{\tau})\int_0^{\bar{\tau}}(\bar{\tau}-2\vartheta)T_2(\bar{\tau}-\vartheta)T_1'(\vartheta)\,d\vartheta d\bar{\tau}-\int_0^{\tau}T_2(\tau-\bar{\tau})\int_0^{\bar{\tau}}(\bar{\tau}-2\vartheta)T_2(\bar{\tau}-\vartheta)T_1(\vartheta)\,d\vartheta d\bar{\tau}; \tag{9'}$$

$$\psi_1(\tau)=\int_0^{\tau}T_2(\tau-\bar{\tau})\int_0^{\bar{\tau}}(\bar{\tau}-2\vartheta)T_2(\bar{\tau}-\vartheta)T_1(\vartheta)\,d\vartheta d\bar{\tau}; \tag{9''}$$

$$\varphi(\tau)=\int_0^{\tau}T_2(\tau-\bar{\tau})\int_0^{\bar{\tau}}T_1(\bar{\tau}-\vartheta)T_2(\vartheta)\,d\vartheta d\bar{\tau}; \tag{9'''}$$

$$d=\frac{r_1^{n_0-l_0+2}}{a_0(n_0-l_0+2)^2}.$$

From relation (9), for the case under consideration, on the basis of measurements of the values of the temperatures (or other potentials), we can determine at the points $r = 0$ and $r = r_1$ the values of the parameters ν and d. Since these two parameters are constant, each of them individually can be determined either in a single realization on the basis of measurements of T_1 and T_2 or in two with different laws of the thermal effect. From differentiation of (9), we have

$$d=\frac{\psi_\tau'\psi_1-\psi\psi_{1\tau}'}{\varphi_\tau'\psi_1-\varphi\psi_{1\tau}'},\quad \nu=\frac{\psi_{1\tau}'\varphi-\psi\varphi_\tau'}{\psi_{1\tau}'\varphi-\psi_1\varphi_\tau'}=\frac{\psi-d\varphi}{\psi_1}. \tag{10}$$

Moreover, the following relations hold:

$$d=\frac{\psi_i\psi_{1j}-\psi_j\psi_{1i}}{\varphi_i\psi_{1j}-\varphi_j\psi_{1i}};\quad \nu=\frac{\psi_i-d\varphi_i}{\psi_{1i}}=\frac{\psi_1-d\varphi_j}{\psi_{1j}}, \tag{11}$$

where the subscripts i, j relate to different time intervals of a single realization or to arbitrary ones of two different realizations. The relations obtained do not, however, enable us to determine individually all the desired parameters. In order to obtain this information, we must evidently have the results of temperature measurements not only at $r = 0$ and $r = r_1$ but also at some point $r = r_2$. Then from (9), after determining the parameters ν and d by one of the methods mentioned above, we obtain

$$\frac{(\psi-\nu\psi_1)_{r=r_1}}{(\psi-\nu\psi_1)_{r=r_2}}=\left(\frac{r_1}{r_2}\right)^{n_0-l_0+2}\frac{\varphi_{r=r_1}}{\varphi_{r=r_2}}, \tag{12}$$

from which it follows that:

$$l_0=1+k-\nu\ln\left[\frac{(\psi-\psi_1)_{r=r_1}}{(\psi-\psi_1)_{r=r_2}}\right]\Big/\ln\left(\frac{r_1}{r_2}\right); \tag{13}$$

$$n_0 = k - 1 + (1 - \nu) \ln \left[\frac{(\psi - \psi_1)_{r=r_1}}{(\psi - \psi_1)_{r=r_2}} \frac{\varphi_{r=r_2}}{\varphi_{r=r_1}} \right] \Big/ \ln \left(\frac{r_1}{r_2} \right); \quad (14)$$

$$a_0 = \frac{r_1^{n_0 - l_0 + 2}}{(n_0 - l_0 + 2)^2 d_{r=r_1}} = \frac{r_2^{n_0 - l_0 + 2}}{(n_0 - l_0 + 2)^2 d_{r=r_2}}. \quad (15)$$

It can be shown that for other relative locations of the measurement points as well, and furthermore, for other laws of variation of λ and c with respect to the spatial variable, we can construct exact explicit functions of various degrees of complexity. The essence of the method of construction of such functions consists in finding in three-dimensional space some Laplace mappings of differential equations in the transform parameter, one of whose particular solutions is the relation of the mappings of the temperatures at some points in terms of a coordinate which in turn is expressed by means of some special functions whose arguments contain the desired coefficients.

NOTATION

τ, $\bar{\tau}$, ϑ, time; r, coordinate; T, temperature; λ, thermal conductivity; c, volumetric heat capacity.

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NONSTATIONARY HEAT TRANSFER BY THE METHOD OF SOLVING THE INVERSE HEAT-CONDUCTION PROBLEM

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The heat-transfer coefficient between a gas and a solid under nonstationary conditions is investigated and computational dependences are obtained.

In multimode, pulse power plants of short operating time an important role is played by transients that are characterized by a gasdynamic and thermal nonstationarity. As investigations showed, the thermogasdynamic nonstationarity can be manifest during a large or even the whole operating time.

The thermal state of a construction using transition regimes has been studied inadequately. The mean temperatures, as well as the temperature fields determined by means of the averaged parameters, do not reflect the features of the transients and do not satisfy the requirements of practice [1].

In the general engineering [2, 3] and specialized [4, 5] purpose papers, the heat-transfer coefficient is averaged, as a rule. Insertion of the quasistationary heat-transfer coefficients does not reduce the problem of nonstationary heat transfer. Unfortunately, there are no relationships on the change in the heat-transfer coefficient in time under transient heat-transfer conditions.

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